Midterm Exam
Math 34B
November 2007

Your name:
Your perm. #:
Your signature:

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Total (100):

When presenting your solutions, be sure to write down all the steps clearly and cleanly!

If you don't show work, and only write down your final result for a problem, you'll receive no credit for that problem!

1. (25 points) A disease among bees is discovered at the beginning of Oct. 1. The rate at which bees are being killed by the disease is $r(t) = t^2 + 10t + 20$ bees per hour t hours after the beginning of October 1.
1) How many bees are killed during the first day of October?
2) How many bees are killed during the third day of October?
3) How many bees are killed in the second week of October?

Note: there is no need to obtain numbers in decimal form, an anwser in the form of say $40^3 + 10 \cdot 20^2$ is sufficient. But you can not lease an answer in the form of an integral. You have to calculate the integrals.

Solution 1) This is given by (since there are 24 hours in each day)

$$\int_0^{24} r(t)dt = \int_0^{24} (t^2 + 10t + 20)dt = [\frac{1}{3}t^3 + 5t^2 + 20t]_0^{24}$$
$$= \frac{1}{3}24^3 + 5 \cdot 24^2 + 20 \cdot 24 = 8 \cdot 24^2 + 5 \cdot 24^2 + 20 \cdot 24 = 7968.$$
$$(1)$$

2) This is given by (since there are 24 hours in each day)

$$\int_{48}^{72} r(t)dt = [\frac{1}{3}t^3 + 5t^2 + 20t]_{48}^{72}$$
$$= [\frac{1}{3}72^3 + 5 \cdot 72^2 + 20 \cdot 72] - [\frac{1}{3}48^3 + 5 \cdot 48^2 + 20 \cdot 48]. \quad (2)$$

3) This is given by (since there are 7 days in each week and 24 hours in each day)

$$\int_{7 \cdot 24}^{14 \times 24} r(t)dt = [\frac{1}{3}t^3 + 5t^2 + 20t]_{7 \cdot 24}^{14 \cdot 24}$$
$$= [\frac{1}{3}7^3 \cdot 24^3 + 5 \cdot 7^2 \cdot 24^2 + 20 \cdot 7 \cdot 24] - [\frac{1}{3}14^3 \cdot 24^3 + 5 \cdot 14^2 \cdot 24^2 + 20 \cdot 14 \cdot 24].$$
$$(3)$$

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2. (25 points) The table shows the rate at which chloride is put into a container of water in *gram/hour*:

time	1:00	2:00	4:00	5:00	5:30	6:30	7:00	8:00	9:00
rate	10	5	10	5	10	5	10	5	10

Approximately how much chloride is put into the container between $2:00$ and $8:00$? (It's important for you to write down a Riemann sum for this problem, and then calculate the number you need. Beware that the time intervals are not all the same length. You can use either the left end point Riemann sum or the right end point Riemann sum. But don't use any other method.)

Solution

The left endpoint Riemann sum

If we use the left endpoint Riemann sum, then the desired amount of chloride is given by

$$5 \cdot (4 - 2) + 10 \cdot (5 - 4) + 5 \cdot (5.5 - 5) + 10 \cdot (6.5 - 5.5) + 5 \cdot (7 - 6.5) + 10 \cdot (8 - 7)$$

$$= \quad 10 + 10 + 2.5 + 10 + 2.5 + 10 = 45. \tag{4}$$

So approximately 45 grams of chloride is put into the container between $2:00$ and $8:00$.

The right endpoint Riemann sum

If we use the right endpoint Riemann sum, then the desired amount is given by

$$10 \cdot (4 - 2) + 5 \cdot (5 - 4) + 10 \cdot (5.5 - 5) + 5 \cdot (6.5 - 5.5) + 10 \cdot (7 - 6.5) + 5 \cdot (8 - 7)$$

$$= \quad 20 + 5 + 5 + 5 + 5 + 5 = 45. \tag{5}$$

This happens to be the same as the above answer.

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3. (25 points) Find the following five integrals

$$\int (2x^2 - 3x + 3)dx, \quad \int_0^3 e^{2t}dt, \quad \int_0^2 2^x dx, \quad \int_0^{\frac{\pi}{4}} 3\sin(2t)dt, \quad \int_1^6 (f'(t) + t)dt,$$

where $f(t)$ is a function with values $f(1) = -2$ and $f(6) = -12$.

Solution

1)

$$\int (2x^2 - 3x + 3)dx = \frac{2}{3}x^3 - \frac{3}{2}x^2 + 3x + C. \tag{6}$$

2)

$$\int_0^3 e^{2t}dt = [\frac{1}{2}e^{2t}]_0^3 = \frac{1}{2}e^6 - \frac{1}{2}. \tag{7}$$

3)

$$\int_0^2 2^x dx = [\frac{1}{\ln 2}2^x]_0^2 = \frac{1}{\ln 2}2^2 - \frac{1}{\ln 2} = \frac{3}{\ln 2}. \tag{8}$$

4)

$$\int_0^{\frac{\pi}{4}} 3\sin(2t)dt = [-\frac{3}{2}\cos(2t)]_0^{\frac{\pi}{4}}$$
$$= -\frac{3}{2}[\cos\frac{\pi}{2} - \cos 0] = -\frac{3}{2}[0 - 1] = \frac{3}{2}. \tag{9}$$

5)

$$\int_1^6 (f'(t) + t)dt = [f(t) + \frac{1}{2}t^2]_1^6 = [f(6) + \frac{1}{2}6^2] - [f(1) + \frac{1}{2}]$$
$$= [-12 + 18] - [-2 + \frac{1}{2}] = 6 + \frac{3}{2} = 6\frac{2}{3}. \tag{10}$$

4. (25 points) The number of crocodiles in a wild animal park increases at the rate of 6 per day. Their appetite is also on the rise. The number of frogs each crocodile eats in one day increases at a rate of 3 per day.

1) Let $m(t)$ denote the number of frogs being eaten by all the crocodiles in the park in the t-th day. How fast is $m(t)$ increasing?

2) Assume that at the start of the week, there were 20 crocodiles and each crocodile ate 10 frogs a day. How many frogs are eaten by all the crocodiles in the park in the last day of the week?

3) (extra credit) Assume there were 1,000 frogs at the beginning of the week. How many frogs are left at the end of the week? (We assume that no frog is born in the week, and no frog dies or escapes except being eaten by crocodiles.)

Solution

1) Let $N(t)$ denote the number of crocodiles in the park on the t-th day, and $E(t)$ the number of frogs eaten by each crocodile in the t-th day. Then we have

$$m(t) = N(t) \cdot E(t). \tag{11}$$

By the given information we have $N'(t) = 6$ and $E'(t) = 3$. By the product rule we have

$$m'(t) = N'(t) \cdot E(t) + N(t) \cdot E'(t) = 6E(t) + 3N(t). \tag{12}$$

So $m(t)$ is increasing at the rate $6E(t) + 3N(t)$. (For example, in the situation of 2), we have $m'(0) = 6 \cdot 10 + 3 \cdot 20 = 120$, i.e. $m(t)$ is increasing at the rate 120 frogs per day at the start of the week.)

2) This is given by

$$\int_6^7 m(t)dt. \tag{13}$$

Indeed, $m(t)$ is the rate of change for the function $M(t)$, which is defined to be the number of frogs eaten by all crocodiles in the park up to time t. We can compute this integral precisely as in 3) below. But we can also make an

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approximation

$$\int_6^7 m(t)dt \approx m(6) \cdot (7-6) = m(6).\tag{14}$$

(If you use $m(7)$, that's also ok.) Since $m(6) = N(6) \cdot E(6)$, we need to find $N(6)$ and $E(6)$. Now the fundamental theorem of calculus tells us

$$N(6) - N(0) = \int_0^6 N'(t)dt = \int_0^6 6dt = 36,$$
$$E(6) - E(0) = \int_0^6 E'(t)dt = \int_0^6 3dt = 18.\tag{15}$$

Since $N(0) = 20$ and $E(0) = 10$ we infer $N(6) = N(0) + 36 = 56$ and $E(6) = E(0) + 18 = 28$. It follows that $m(6) = N(6) \cdot E(6) = 56 \cdot 18 = 1,008$. Thus 1,008 frogs are eaten by all the crocodiles in the park in the last day of the week.

3) By 2) we see that 1,008 frogs should be eaten in the last day of the week. But there were only 1,000 frogs in the beginning. So there aren't enough frogs for the crocodiles to eat. Hence the answer is that there is no frog left at the end of the week.

Actually, we should replace $1,000$ by e.g. $10,000$, which makes more sense. So let's assume that there were $10,000$ frogs at the beginning of the week. We need to find the total number of frogs eaten in the entire week. This is given by

$$\int_0^7 m(t)dt.\tag{16}$$

Arguing as in 2), we find

$$N(t) = N(0) + \int_0^t N'(t)dt = 20 + 6t, E(t) = E(0) + \int_0^t E'(t)dt = 10 + 3t.\tag{17}$$

It follows that

$$m(t) = N(t) \cdot E(t) = (20 + 6t)(10 + 3t) = 200 + 120t + 18t^2,\tag{18}$$

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$$\int_0^7 m(t)dt = \int_0^7 (200 + 120t + 18t^2)dt = [200t + 60t^2 + 6t^3]_0^7$$
$$= 200 \cdot 7 + 60 \cdot 7^2 + 6 \cdot 7^3 = 1400 + 2940 + 2058 = 6398.$$

(19)

Finally, the number of frogs left at the end of the week is given by $10,000 - 6,398 = 3,602$. Hence there are $3,602$ frogs left at the end of the week.