Math 108B - Home Work # 6

Due: Wednesday, June 4, 2008

1. If A is an $n \times n$ upper-triangular matrix (i.e., $A_{ij} = 0$ for all $i > j$), show that $\det A = \prod_{i=1}^{n} A_{ii}$. (You may use either the definition of determinant given in class, or else the standard definition for matrices.)

2. Let A be a nilpotent $n \times n$ matrix. Show that A is diagonalizable if and only if $A = 0$.

3. This question asks you to find some 3×3 matrices. Your answers will be non-diagonalizable, since they will each have only 2 linearly independent eigenvectors.

   a) Give an example of a 3×3 matrix with only one eigenvalue (over $\mathbb{C}$), but with a 2-dimensional eigenspace. What are the generalized-eigenspaces of $\mathbb{C}^3$ for your example?

   b) Give an example of a 3×3 matrix with only two distinct eigenvalues (over $\mathbb{C}$), each of which has a 1-dimensional eigenspace. What are the generalized eigenspaces of $\mathbb{C}^3$ for your example?

4. LADR Exercises: p. 188-190: 3, 5, 10, 11

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