Math 108B - Home Work # 3

Due: Wednesday, April 23, 2008

1. LADR p. 123-124: Exercises 10, 14, 15, 21 (For 14, first read Corollary 6.27).

2. (Do not turn in) For Ex. 10, does anything change if you apply the Gram-Schmidt Process to the basis $\{1, x, x^2\}$ for $\mathcal{P}_2(\mathbb{C})$ with the inner product $\langle p, q\rangle = \int_0^1 p(x)\overline{q(x)}\, dx$?

3. If U is a *subset* of an inner product space V (but not necessarily a subspace), we can still define
$$U^\perp = \{v \in V \mid \langle v, u\rangle = 0\ \forall u \in U\}.$$
(a) Prove that $U^\perp = \text{span}(U)^\perp$.

   (Recall, that span(U) is the subspace of V consisting of all finite F-linear combinations of vectors in U.)

   (b) Use (a) to prove that $(U^\perp)^\perp = \text{span}(U)$.

   In particular, this exercise implies that if $\{u_1, \ldots, u_m\}$ is a basis for the subspace U, then
$$U^\perp = \{v \in V \mid \langle v, u_i\rangle = 0\ \forall i\ \}.$$