Math 108B - Home Work # 2

Due: Wednesday, April 16, 2008

1. Let $b_1, \ldots, b_n$ be positive real numbers. Check that the form

$$\langle z, w\rangle = b_1 z_1 \bar{w}_1 + \cdots b_n z_n \bar{w}_n$$

defines an inner product on F^n, where $z = (z_1, \ldots, z_n)$ and $w = (w_1, \ldots, w_n)$. (In particular, the dot product on $\mathbb{C}^n$ is an inner product.)

2. Let V be an F-vector space with basis $\{v_1, \ldots, v_n\}$, and let $B = (b_{ij})$ be the $n \times n$ matrix with entries $b_{ij} = \langle v_i, v_j\rangle \in F$. Show that
   (a) $b_{ii} > 0$ for $1 \leq i \leq n$; and
   (b) $B = \bar{B}^t$, i.e., $b_{ij} = \bar{b}_{ji}$ for all $1 \leq i, j \leq n$.

3. Give an example of a 2×2 matrix B satisfying (a) and (b) above that does not define an inner product on F^2 with $\langle e_i, e_j\rangle = b_{ij}$ for $1 \leq i, j \leq 2$. ($\{e_1, e_2\}$ is the standard basis for F^2.) Hint: Construct the matrix B so that there is a vector v whose norm would be negative with respect to the corresponding inner product.

4. Problems 4, 5 and 6 on p. 122 of LADR.

5. (Bonus) Let $x_1, \ldots, x_n$ be positive real numbers. Prove that

$$(x_1 + \cdots + x_n)\left(\frac{1}{x_1} + \cdots + \frac{1}{x_n}\right) \geq n^2.$$

(Hint: Use the Cauchy-Schwarz inequality.)