MATH 108 A WINTER 2008 FINAL EXAM

$NAME:$

SHOW YOUR WORK CLEARLY. OTHERWISE NO PARTIAL CREDIT.

FINAL (40) _______ _____

MIDTERM (20) ____ _____

Attendance+ quizzes (30) _______

HOMEWORK (15) _______ ____

FINAL GRADE ____ _______ _____

1) (6 points) Complete the following definitions: (V vector space)

(a) $\vec{v}_1, \vec{v}_2, .., \vec{v}_k \in V$ are linearly independent if...

(b) Let $T : V \to V$ be a linear map. $\lambda \in \mathbb{C}$ is an eigenvalue of T if.................

....

2) (10 points) Find a basis of the subspace

(a) $U_1 = \{\vec{x} = (x_1, .., x_6) \in \mathbb{R}^6 \, : \, x_1 = -x_3, \; x_4 - 2x_6 = 0\}.$

(b) $U_2 = span\{\vec{v}_1, \, \vec{v}_2, \, \vec{v}_3, \, \vec{v}_4\}$, with
$\vec{v}_1 = (1, -2, -3, 0), \; \vec{v}_2 = (0, 0, 1, 0), \; \vec{v}_3 = (-1, 2, 1, -1), \; \vec{v}_4 = (0, 0, 1, 1).$

3) (15 points) Given the linear map $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined as

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4x - 5y \\ 2x - 2y \\ x + y \end{pmatrix}.$$

(a) Find the matrix A_T which represents this linear map.
(b) Describe the $null(T)$ and the $Range(T)$.
(c) Is T injective? Surjective?
(d) Find the eigenvalues and its associated eigenvectors.
(e) Compute the matrix A_T^{100}.

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4) (12 points) Let V be a vector space and $T \in \mathcal{L}(V)$ such that $T^3 = T$.
a) Which are the possible eigenvalues of T?
b) Prove that $range(T) \cap null(T) = \{0\}$.
Assume now that V is finite dimensional with $dim(V) = n$.
c) Prove that $null(T) \oplus range(T) = V$.
d) Give an example of a 3×3 non-diagonal matrix A that verify the equation $A^3 = A$.

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