Math 108A - Home Work # 3
Due: April 23, 2008

1. Exercises 8, 9, 12, 14 on p. 35-36 in LADR.

2. Let $v_1, \ldots, v_m$ and u be vectors in a vector space V. Show that

$$u \in span(v_1, \ldots, v_m) \iff span(v_1, \ldots, v_m, u) = span(v_1, \ldots, v_m).$$

3. (a) Prove that $\{v_1, \ldots, v_m\}$ is a linearly independent set of vectors if and only if any $u \in span(v_1, \ldots, v_m)$ can be written uniquely as a linear combination $u = c_1 v_1 + \cdots + c_m v_m$ for scalars $c_1, \ldots, c_m \in F$.

   (b) Prove that $\{v_1, \ldots, v_m\}$ is a linearly independent set of vectors if and only if

$$span(v_1, \ldots, v_m) = Fv_1 \oplus Fv_2 \oplus \cdots \oplus Fv_m.$$

   (Note: by definition $span(v_1, \ldots, v_m) = Fv_1 + Fv_2 + \cdots + Fv_m$.)