

Math 108A - Home Work # 2

Due: April 18, 2008

1. In class, we saw that the set $\mathbb{R}^{\mathbb{R}}$ of all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ is an \mathbb{R} -vector space (with the 0-function $0(x) = 0 \forall x \in \mathbb{R}$ as the 0-vector). Which of the following subsets of $\mathbb{R}^{\mathbb{R}}$ are subspaces? Justify your answers.

(a) $\mathcal{C} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid f \text{ is continuous} \}$

(b) $\mathcal{D} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid f \text{ is differentiable} \}$

(c) $\mathcal{E} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid f(0) = 1 \}$

(d) $\mathcal{F} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid f(0) = 0 \}$

(e) $\mathcal{G} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid \forall x \in \mathbb{R} f(x) \neq 0 \}$

(f) $\mathcal{B} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid \exists M \in \mathbb{R} \forall x \in \mathbb{R} |f(x)| \leq M \}$

(g) $\mathcal{L} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid \lim_{x \rightarrow \infty} f(x) = 0 \}$

2. Problems 5, 9, 13, 15 on p. 19-20 in LADR.