Name: ____________

Perm No.: ____________

Math 108A - Final Exam

June 10, 2008

Instructions:

- This exam consists of 5 problems for a total of 65 points.
- You must show all your work and fully justify your answers in order to receive full credit.
- If your justification involves a result proved in class, you should summarize what that result says and, if necessary, explain how you are using it.
- Partial credit will be given for work that is relevant and correct.
- You may assume the results of earlier questions are true, even if you can't prove them, in order to do later questions (eg. to do part (c), you may assume parts (a) and (b)).
- Your proofs will be graded for clarity and organization, in addition to correctness.
- No books, notes or calculators are allowed.
- Write your answers on the test itself, in the space alotted. Scratch paper is available if you need it. You may want to work out your solutions first on scratch paper, so that you can write them on the test as neatly as possible. You may attach additional pages if necessary.

1	
2	
3	
4	
5	
Total	

1. (10 points) Let $V = span\{(1,-1,0,0),\ (2,0,2,1),\ (0,2,2,1),\ (-1,-1,-2,-1)\} \subseteq F^4$. What is $\dim V$? Justify your answer.

2. Let $U = \{(x, y, z) \in F^3 \mid z = 3x + y\}$

   (a) (4 points) Find a linear map $T : F^3 \to F$ such that $U = null(T)$.

   (b) (6 points) Find a basis for U.

3. (10 points) Let $T \in \mathcal{L}(V)$ for a finite-dimensional vector space V. Show that T is invertible if and only if 0 is not an eigenvalue of T.

4. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (y + z, x + z, x + y)$.

   (a) (3 points) Find the matrix of T with respect to the standard basis.

   (b) (6 points) Is T invertible? Justify your answer.

(c) (5 points) What are the eigenvalues of T?

(d) (6 points) Describe the eigenspaces of T for each eigenvalue.

(e) (5 points) Find a basis of $\mathbb{R}^3$ consisting of eigenvectors of T, or show that no such basis exists.

5. Let $v = (c_1, \ldots, c_n) \neq 0 \in F^n$ and let $S : F^n \to F^n$ be the unique linear map such that $S(e_i) = v$ for each standard basis vector e_i. (The matrix of S in the standard basis has v as each column.)

   (a) (3 points) Describe $null(S)$.

   (b) (2 points) Show that v is an eigenvector of S. What is the corresponding eigenvalue?

(c) (5 points) Prove that S is diagonalizable if and only if $c_1 + \cdots + c_n \neq 0$.