Math 108A - Home Work # 1

Due: April 8, 2009

1. For any $z \in \mathbb{C}$, prove that $z \in \mathbb{R}$ if and only if $\overline{z} = z$.

2. Is the set $\mathbb{Z}$ of integers (with the usual operations of addition and multiplication) a vector space? Why or why not?

3. Consider the set $V = \{(x, y) \in \mathbb{R}^2 \mid x, y \geq 0\}$ consisting of all vectors in the first quadrant of $\mathbb{R}^2$ (considered with usual vector addition and scalar multiplication). Which vector space axioms (as listed on p. 9) hold for V and which fail? Justify your answers.

4. Let $\mathcal{P}(\mathbb{R})$ denote the set of all polynomials in the variable x with real coefficients. Show that $\mathcal{P}(\mathbb{R})$ is a vector space over $\mathbb{R}$. (You should *briefly* justify/check each of the axioms.)

5. Let V be a vector space over F. In class we saw that any vector v has a unique additive inverse, denoted $-v$.

   (a) Using only the vector space axioms, show that for any $v \in V$, the additive inverse of v is given by $-1 \cdot v$. Mention which axiom you are using in each step of the proof. (Thus, we now know that $-v = -1 \cdot v$ for any vector $v \in V$.)

   (b) Let V be a vector space over F. Show that $-(-v) = v$ for any $v \in V$. Again, mention which axioms or previously proved results you are using in each step.