Math 108A - Home Work # 7

Due: May 27, 2009

1. For the following matrices:

   (a) Find the eigenvalues (over $F = \mathbb{C}$).

   (b) Describe the eigenspace for each eigenvalue. (i.e., describe all the eigenvectors for each eigenvalue).

   (c) Determine whether F^2 or F^3 has a basis consisting of eigenvectors of the matrix.

$$A = \begin{pmatrix} 1 & 1 \\ -2 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

2. $\lambda = 2$ is an eigenvalue of the matrix

$$A = \begin{pmatrix} 4 & -12 & -6 \\ 1 & -4 & -3 \\ -1 & 6 & 5 \end{pmatrix}.$$

   (a) Find a basis for the eigenspace of A for the eigenvalue $\lambda = 2$.

   (b) Does A have any other eigenvalues? If so, find them and find corresponding eigenvectors.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation given by the matrix

$$\begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 0 & 3 \end{pmatrix}$$

   with respect to the standard bases. Find bases for $\mathbb{R}^2$ and $\mathbb{R}^3$ in which the matrix of T is

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

4. Let A be an $m \times n$ matrix with entries in F. The different row operations that can be performed on A are

   - $R1(i, a)$: Multiply the i^{th} row of A by a nonzero scalar $a \in F$. $(1 \leq i \leq m)$
   - $R2(i, j)$: Swap the i^{th} and j^{th} rows of A. $(1 \leq i, j \leq m)$
   - $R3(i, j)$: Add the i^{th} row of A to the j^{th} row of A. $(1 \leq i, j \leq m)$

   For each row operation R listed above, exhibit an $m \times m$ matrix X such that XA is the matrix obtained by applying the row operation R to A. Verify that each such X is invertible. (Hint: think about which row operation would undo R.)