



Figure 7: Repeated transitions between the torus state and the dynamic parallel state for a modified version of the three-dimensional schooling model of Couzin *et al.* (12) with $r_r = 1$, $\Delta r_o = 2$, and $\Delta r_a = 9$. We modify (12) by considering rare but substantial variations in the headings of individuals obtained from the deterministic algorithm. Specifically, each agent changes the sign of its desired direction with probability $p = 0.005$, with the new direction chosen according to a spherically wrapped Gaussian distribution with standard deviation $\sigma = 5$. (This differs from (12) for which $p = 1$ and $\sigma = .05$, so that all agents modify their direction at each timestep with small variation.) Two coarse variables measure the collective behavior (12): (dashed) $R(t)$ = angular momentum, (solid) $S(t)$ = group polarization; both are normalized so that they reach their maximum value at 1 and minimum at 0.