MATH 115B PROBLEM SET VI
JUNE 5, 2007

(1) Show that there are infinitely many even integers n with the property that both $n+1$ and $(n/2)+1$ are perfect squares. Exhibit two such integers.

(2) Find the fundamental solutions of the following equations:
(i) $x^2 - 29y^2 = 1$;
(ii) $x^2 - 41y^2 = 1$.

(3)(a) Prove that whenever the equation $x^2 - dy^2 = c$ is soluble, then it has infinitely many solutions.
[Hint: If u, v satisfy $x^2 - dy^2 = c$ and r, s satisfy $x^2 - dy^2 = 1$, then

$$(ur \pm dvs)^2 - d(us \pm vr)^2 = (u^2 - dv^2)(r^2 - ds^2) = c.]$$

(b) Given that $x = 16$, $y = 6$ is a solution of $x^2 - 7y^2 = 4$, find two other positive solutions.