

Math 34B Spring Quarter Midterm Examination
May 4, 2006

NAME: Answer Key (ii)

TA & DISCUSSION SECTION:

You have 70 minutes in which to complete this examination. Attempt all of the questions. Note that you will not be awarded full credit on a question unless your answer is clearly, carefully and neatly stated.

UP TO 5 BONUS POINTS (ADDED DIRECTLY TO YOUR SCORE ON THE EXAMINATION) WILL BE AWARDED FOR NEATLY AND CAREFULLY PRESENTED WORK!

Problem	Maximum Score	Score
1	9	
2	9	
3	9	
4	9	
5	6	
NEATNESS BONUS	5	
Total	42	

(1) Find the derivatives with respect to x of the following functions:

(i) $f(x) = 9 \sin(3x) + 3x + 1$

$$\begin{aligned} f'(x) &= 3 \cdot 9 \cos(3x) + 3 \\ &= 27 \cos(3x) + 3 \end{aligned}$$

(ii) $f(x) = e^{3x} \sin(5x)$

$$\begin{aligned} f'(x) &= \frac{d}{dx} (e^{3x}) \sin(5x) + e^{3x} \frac{d}{dx} (\sin(5x)) \\ &= 3e^{3x} \sin(5x) + 5 \cdot e^{3x} \cos(5x) \\ &= e^{3x} (3 \sin(5x) + 5 \cos(5x)) \end{aligned}$$

(iii) $y = (g(x))^2$

$$y = g(x) \cdot g(x)$$

$$\begin{aligned} \therefore \frac{dy}{dx} &= g'(x) \cdot g(x) + g(x) \cdot g'(x) \\ &= 2g(x) \cdot g'(x). \end{aligned}$$

(2) Find the following integrals:

(i) $\int 3e^{9t} dt$

$$\int 3e^{9t} dt = \frac{1}{3} e^{9t} + C$$

(ii) $\int_1^3 x^3 dx$

$$\begin{aligned} \int_1^3 x^3 dx &= \left[\frac{x^4}{4} \right]_1^3 \\ &= \frac{3^4}{4} - \frac{1^3}{4} \\ &= \frac{80}{4} \\ &= 20 \end{aligned}$$

(iii) $\int_1^{\pi} 3 dx$

$$\begin{aligned} \int_1^{\pi} 3 dx &= \left[3x \right]_1^{\pi} \\ &= 3\pi - 3. \end{aligned}$$

(3) Consider the function $f(t) = 5 \cos(3t + 3)$, where t denotes time measured in seconds.

(a) What is the period of $f(t)$?

The period of $f(t)$ is $\frac{2\pi}{3}$ seconds.

(b) What is the frequency of $f(t)$?

The frequency of $f(t)$ is $\frac{3}{2\pi}$ Herz.

(c) What is the amplitude of $f(t)$?

The amplitude of $f(t)$ is equal to 5.

(4) The height above the ground in metres of a guided missile t seconds after being launched is $h(t) = 9t + 3t^2$.

(a) What is the velocity of the missile after t seconds?

Let $v(t)$ = velocity of the missile after t seconds.

Then

$$\begin{aligned} v(t) &= \frac{dh}{dt} = \frac{d}{dt} (9t + 3t^2) \\ &= 9 + 6t \quad \text{m/s.} \end{aligned}$$

(b) How many seconds after the launch is the velocity of the missile equal to 45 m/s?

The velocity of the ~~missile~~ ^{missile} is 45 m/s when

$$\begin{aligned} 9 + 6t &= 45 \\ \therefore 6t &= 45 - 9 \\ &= 36 \\ \therefore t &= 6 \text{ seconds.} \end{aligned}$$

Hence the velocity of the missile is equal to 45 m/s after 6 seconds.

(c) What is the acceleration of the missile t seconds after being launched?

Let $a(t)$ = acceleration of the missile after t seconds.

$$\begin{aligned} \text{Then} \quad a(t) &= \frac{dv}{dt} \\ &= \frac{d}{dt} (9 + 6t) \\ &= 6 \text{ m/s}^2. \end{aligned}$$

(5) The radius of a circular tide pool is increasing at a rate of 1 metre per second. At time $t = 0$, the area of the tide pool is equal to $9\pi \text{ m}^2$.

(a) Find the radius of the tide pool after t seconds.

Let $r(t)$ = radius of tide pool after t seconds

$$\text{Then } \frac{dr}{dt} = 1, \text{ and so } r(t) = \int 1 \cdot dt = t + C$$

When $t=0$, the radius of the tide pool is 3m (since its area is $9\pi \text{ m}^2$). So

$$3 = 0 + C,$$

and therefore $C = 3$.

$$\therefore r(t) = t + 3 \text{ metres.}$$

(b) Find the rate at which the area of the tide pool is increasing after t seconds.

Let $A(t)$ = area of the tide pool after t seconds.

$$\begin{aligned} \text{Then } A(t) &= \pi r(t)^2 \\ &= \pi (t+3)^2 \end{aligned}$$

$$\therefore \frac{dA}{dt} = 2\pi (t+3) \text{ m}^2/\text{s}$$

Hence the area of the tide pool is increasing at a rate of $2\pi (t+3) \text{ m}^2/\text{s}$ after t seconds.