Math 34B Spring Quarter Midterm Examination
May 4, 2006

NAME: Answer Key (ii)

TA & DISCUSSION SECTION: .....

You have 70 minutes in which to complete this examination. Attempt all of the questions.

Note that you will not be awarded full credit on a question unless your answer is clearly, carefully and neatly stated.

UP TO 5 BONUS POINTS (ADDED DIRECTLY TO YOUR SCORE ON THE EXAMINATION) WILL BE AWARDED FOR NEATLY AND CAREFULLY PRESENTED WORK!

Problem	Maximum Score	Score
1	9	
2	9	
3	9	
4	9	
5	6	
NEATNESS BONUS	5	
Total	42	

(1) Find the derivatives with respect to x of the following functions:

(i) $f(x) = 9\sin(3x) + 3x + 1$

$$f'(x) = 3 \cdot 9 \cos(3x) + 3$$
$$= 27 \cos(3x) + 3$$

(ii) $f(x) = e^{3x}\sin(5x)$

$$f'(x) = \frac{d}{dx}\left(e^{3x}\right) \sin(5x) + e^{3x} \frac{d}{dx}\left(\sin(5x)\right)$$
$$= 3e^{3x} \sin(5x) + 5 \cdot e^{3x} \cos(5x)$$
$$= e^{3x}\left(3 \sin(5x) + 5 \cos(5x)\right)$$

(iii) $y = (g(x))^2$

$$y = g(x) \cdot g(x)$$
$$\therefore \quad \frac{dy}{dx} = g'(x) \cdot g(x) + g(x) \cdot g'(x)$$
$$= 2\, g(x) \cdot g'(x).$$

(2) Find the following integrals:

(i) $\int 3e^{9t}\,dt$

$$\int 3e^{9t}\,dt = \frac{1}{3}e^{9t} + C$$

(ii) $\int_1^3 x^3\,dx$

$$\int_1^3 x^3\,dx = \left[\frac{x^4}{4}\right]_1^3$$

$$= \frac{3^4}{4} - \frac{1^3}{4}$$

$$= \frac{80}{4}$$

$$= 20$$

(iii) $\int_1^\pi 3\,dx$

$$\int_1^\pi 3\,dx = \left[3x\right]_1^\pi$$

$$= 3\pi - 3.$$

(3) Consider the function $f(t) = 5\cos(3t+3)$, where t denotes time measured in seconds.

(a) What is the period of $f(t)$?

The period of $f(t)$ is $\frac{2\pi}{3}$ seconds.

(b) What is the frequency of $f(t)$?

The frequency of $f(t)$ is $\frac{3}{2\pi}$ Herz.

(c) What is the amplitude of $f(t)$?

The amplitude of $f(t)$ is equal to 5.

(4) The height above the ground in metres of a guided missile t seconds after being launched is $h(t) = 9t + 3t^2$.

(a) What is the velocity of the missile after t seconds?

Let $v(t)$ = Velocity of the missile after t seconds.

Then

$$v(t) = \frac{dh}{dt} = \frac{d}{dt}\left(9t + 3t^2\right)$$

$$= 9 + 6t \quad m/s.$$

(b) How many seconds after the launch is the velocity of the missile equal to $45\,m/s$?

The velocity of the ~~rocket~~ missile is 45 m/s when

$$9 + 6t = 45$$

$$\therefore \quad 6t = 45 - 9$$

$$= 36$$

$$\therefore \quad t = 6 \text{ seconds.}$$

Hence the velocity of the missile is equal to 45 m/s after 6 seconds.

(c) What is the acceleration of the missile t seconds after being launched?

Let $a(t)$ = acceleration of the missile after t seconds.

Then

$$a(t) = \frac{dv}{dt}$$

$$= \frac{d}{dt}\left(9 + 6t\right)$$

$$= 6 \quad m/s^2.$$

(5) The radius of a circular tide pool is increasing at a rate of 1 metre per second. At time $t = 0$, the area of the tide pool is equal to $9\pi\, m^2$.

(a) Find the radius of the tide pool after t seconds.

Let $r(t)$ = radius of tide pool after t seconds

Then $\frac{dr}{dt} = 1$, and so $r(t) = \int 1 \cdot dt = t + C$

When $t=0$, the radius of the tide pool is $3m$ (since its area is $9\pi\, m^2$). So

$$3 = 0 + C,$$

and therefore $C = 3$.

$$\therefore\ r(t) = t + 3 \text{ metres.}$$

(b) Find the rate at which the area of the tide pool is increasing after t seconds.

Let $A(t)$ = area of the tide pool after t seconds.

Then $A(t) = \pi\, r(t)^2$

$$= \pi (t+3)^2$$

$$\therefore\ \frac{dA}{dt} = 2\pi (t+3)\ m^2/s$$

Hence the area of the tide pod is increasing at a rate of $2\pi(t+3)\ m^2/s$ after t seconds.