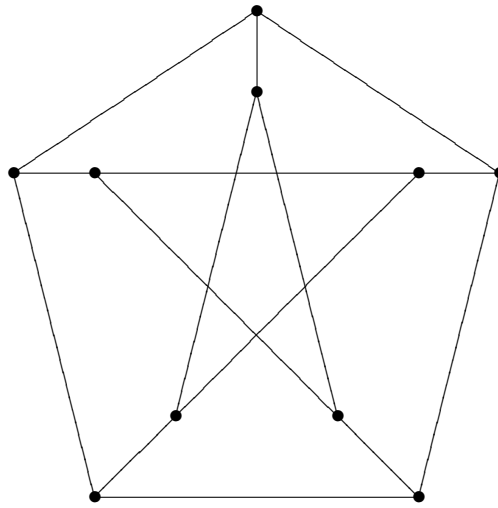


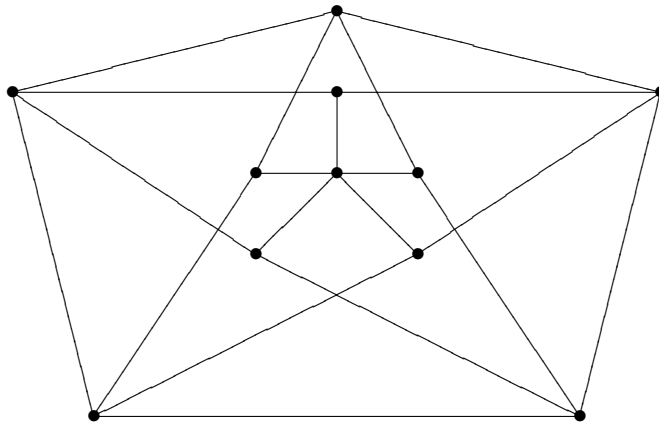
HOMEWORK 4

8 PROBLEMS
DUE: WEDNESDAY, MAY 25, 2011

- (1) Determine the chromatic number of the Petersen graph.



- (2) Determine the chromatic number of the Grötzsch graph.



- (3) Draw a self-dual plane graph on four vertices.
- (4) Draw a self-dual plane graph on seven vertices.
- (5) For a simple connected graph G , with at least two vertices, prove that $\chi(G) = 2$ if and only if G is bipartite.
- (6) For a simple connected graph G , with at least two vertices, prove that $\chi(G) \leq k$ if and only if G is k -partite.
- (7) For a simple connected graph G , with n vertices, prove that $\chi(G) = n$ if and only if $G = K_n$.
- (8) Let G be a simple graph on n vertices and \overline{G} its complement. Show that

$$\chi(G) + \chi(\overline{G}) \geq 2\sqrt{n}.$$