HOMEWORK 2

8 PROBLEMS
DUE: WEDNESDAY, APRIL 27, 2011

(1) Draw two graphs on six vertices that are 1-isomorphic but are not isomorphic.

(2) Let $(\vec{G}, c; s, t)$ be a network and f a flow. Show that if S is a source vertex cut, then $\text{val}(f) = f^+(S) - f^-(S) \leq c^+(S)$.

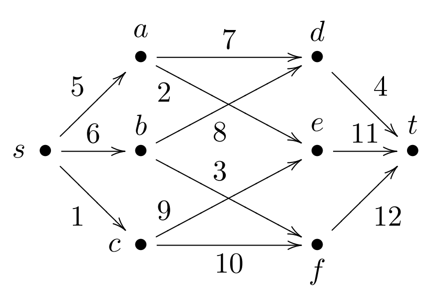
(3) Let $(\vec{G}, c; s, t)$ be a network, f a flow, and p an augmenting path of $\vec{G}$ from s to t with a tolerance of $\delta > 0$. Let f' be given by

$$f'(e) = \begin{cases} f(e) + \delta & \text{if } \eta(e) = \eta(e_i) = (u_{i-1}, u_i), \\ f(e) - \delta & \text{if } \eta(e) = \eta(e_i) = (u_i, u_{i-1}), \\ f(e) & \text{if } e \text{ is not in } p. \end{cases}$$

Show that f' is a flow and $\text{val}(f') = \text{val}(f) + \delta$.

(4) Give an example of a network $\vec{G}$ with a unique maximum flow f.

(5) Use the Ford-Fulkerson Algorithm to find a maximum flow for the network $\vec{G}$ given below. Prove that your flow f is maximum by finding a source vertex cut S such that $\text{val}(f) = c^+(S)$.



(6) Prove Euler's Formula by induction on the number of vertices.

(7) Let G be a plane graph with n vertices, m edges, f faces and k components. Show that

$$n - m + f = k + 1.$$

(8) Let e be an edge of $K_{3,3}$. Show that $K_{3,3} - e$ is planar.