

# HOMEWORK 1

8 PROBLEMS

DUE: WEDNESDAY, APRIL 13, 2011

- (1) Determine all  $m, n \in \mathbb{N}$  such that the complete bipartite graph  $K_{m,n}$  is Hamiltonian.
- (2) Give an example of a strongly connected digraph whose underlying graph is not Hamiltonian.
- (3) Let  $\vec{G}$  be a digraph on  $n \geq 2$  vertices. Let

$$Y = A(\vec{G}) + A(\vec{G})^2 + A(\vec{G})^3 + \cdots + A(\vec{G})^{n-1}.$$

Prove that  $\vec{G}$  is strongly connected if, and only if, all entries of  $Y$  are nonzero.

- (4) Show that there exists a vertex labeling of  $\vec{G}$  such that  $A(\vec{G})$  is a strictly lower triangular matrix if, and only if,  $\vec{G}$  is an acyclic digraph.
- (5) Let  $G$  be a graph with  $n$  vertices, where  $n \geq 2$ . Prove that  $G$  has at least two vertices which are not cut vertices.
- (6) Let  $v$  be a cut vertex of a simple connected graph  $G$ . Prove that  $v$  is not a cut vertex of its complement  $\bar{G}$ .
- (7) Let  $G$  be a simple connected graph with at least two vertices. Prove that

$$\kappa(G) \leq \frac{2m}{n},$$

where  $m$  is the number of edges and  $n$  is the number of vertices.

- (8) Draw the block cut-point graph for the graph  $G$ .

