HOMEWORK 1

(1) Let $A = \{3, 4, 5\}, B = \{3, 4\}, C = \{4\}$. Find $D = A \triangle B \triangle C$.

(2) Suppose 70% of Californians like cheese, 80% like apples and 10% like neither. What percentage of Californians like both cheese and apples?

(3) Use the Principle of Mathematical Induction to prove that for $n \in \mathbb{N}$, $n^3 - n$ is always divisible by 3.

(4) Find a surjective function from $\mathbb{N}$ to $\mathbb{Z}$. Find an injective function from $\mathbb{Z}$ to $\mathbb{N}$.

(5) Write an explicit description of the edgemap for the complete bipartite $(3, 5)$-graph.

(6) Is there a simple graph on 6 vertices such that the vertices all have distinct degree? If not, why not? If so, draw one.

(7) Let G be a k-regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k.

(8) Prove that it is impossible to have a group of nine people at a party such that each one knows exactly five of the others in the group.