University of California, Santa Barbara

Department of Mathematics

Graduate Program Handbook

Revised Fall 2016

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INTRODUCTION

The purpose of this handbook is to detail the requirements of the MA and Ph.D. programs in the Department of Mathematics at the University of California, Santa Barbara. We want you understand exactly what is expected of you as a graduate student in the Mathematics Department. We also include information about issues such as financial support and advising that are very important to the successful completion of your degree. This handbook is for you, so please notify the Staff Graduate Advisor or Faculty Graduate Advisor immediately if you think that something here is unclear or missing. We want your experience as a graduate student to be a fulfilling and rewarding one, so your input is valuable and appreciated.

GRADUATE STUDENT ADVISING

All graduate students will be assigned a faculty advisor upon entering our program. Your faculty advisor is the person you consult on all academic matters of your education. Accordingly, he or she should be consulted before the start of each quarter. When students are more familiar with the department faculty, they may choose their own advisors.

Staff Graduate Advisor

The Staff Graduate Advisor is the person you see to coordinate most of the administrative aspects of your education and employment at the university. This person is your liaison to the Graduate Division and to the Mathematics Department Graduate Committee. The Staff Graduate Advisor is available for assistance with any matter that is affecting your life as a graduate student, and will be able to direct you to various resources and services on campus. You should get to know the Staff Graduate Advisor immediately upon your arrival.

Petitions

Students may petition the Graduate Committee to request an exception to any established departmental policy. Petitions should be drafted by the student after consulting with his or her advisor. This petition process is especially important for incoming students who have done substantial graduate work elsewhere. Such students are encouraged to petition for credit toward our requirements for graduate work done elsewhere.

FINANCIAL MATTERS FOR CONTINUING STUDENTS

Departmental Support

A Ph.D. student *making good progress* can expect to continue to receive financial support. Good progress entails completion of the area requirements before the start of the third year, advancement to candidacy one year thereafter or in any case before the fourth year, and completion of the Ph.D. degree within six years. The schedule may be accelerated by one year for students entering the Ph.D. program with a Master's degree from another institution.

Each student's progress is determined by the Graduate Committee, based on the individual's circumstances, and a formal evaluation is given at least yearly. When the number of graduate students requesting support exceeds the number of available positions, support is allocated competitively on a year-to-year basis with priority given to students fulfilling the criteria above. According to UC policy, students can receive a maximum of 6 years of financial support. The Department of Mathematics awards support in the form of employment as a Teaching Assistant or Teaching Associate. A number of Nonresident Tuition Fellowships are also awarded each year.

If you have questions about your financial support status, please feel free to see the Staff Graduate Advisor or the faculty member serving as Graduate Vice Chair at any time.

Other Employment

Graduate students are also sometimes employed as Readers in the Department of Mathematics. Please note that domestic graduate students may only work more than 50% time at UCSB during the academic year with employment exception approval. International students may never work beyond 50% time during the academic year, according to university policy.

Fellowships

Limited fellowship support is available to continuing students. Refer to Graduate Division web page http://www.graddiv.ucsb.edu/financial to obtain fellowship applications and to find out about fellowship deadlines. Please note that only students who have submitted the Free Application for Federal Student Aid (FAFSA) will be considered for the Graduate Student Fee Fellowships, as well as many of the other campus-wide fellowships.

Need-Based Financial Aid

To apply for need-based financial aid in the form of loans or work-study, you must submit the FAFSA. This application is due March 2 of each year, however students are encouraged to submit them as early as possible. You can submit application materials electronically by visiting the FAFSA website at www.fafsa.gov, or you can obtain the forms from the Student Financial Services

Office on the second floor of the Student Academic and Administrative Services Building (SAASB). All graduate students who are U.S. citizens or Permanent Residents are required to submit the FAFSA on an annual basis.

Fee Offsets

Students who are registered full-time and who meet the following minimum employment criteria will receive a partial fee offset. As of Fall 2016, this amount is $5004 per qualified academic quarter (amount subject to change).

1) 25% time (or greater) appointment as a Graduate Student Researcher for any given academic year quarter; offset is credited at beginning of the quarter of employment. (GSRs hired at 35% or greater qualify for full payment of fees and nonresident tuition when applicable.) This type of appointment is contingent upon availability of faculty funding.

2) 25% time (or greater) appointment as a Teaching Assistant or Teaching Associate for any given academic year quarter; offset is credited at beginning of the quarter of employment.

3) 100 hours (or more) of work completed during the duration of any academic year quarter while employed as a Reader, or a Tutor, or combined employment in these titles; offset is credited at the end of the employment quarter after 100 hours of work have been reported to Payroll.

Graduate Student Health Insurance and UCSB Student Health Service

Payment of graduate student health insurance is mandatory for all graduate students. This requirement can be waived if the student can demonstrate that he or she has coverage from another source as good as or better than that provided through the University of California Student Health Insurance Program (UC SHIP).

The university will provide full payment of health insurance fees for all Teaching Assistants and Teaching Associates employed at 25% or more per quarter, Graduate Student Researchers employed at 25% to 34% per quarter, and Readers or Tutors (or combined employment in these titles) who complete 100 hours (or more) of work during the duration of any academic year quarter. Payment will be made at the beginning of the eligible quarter for TAs, Associates, and GSRs. Health insurance payment for Readers and Tutors will be made at the end of the eligible quarter, after 100 hours of work have been reported to Payroll.

Currently, if the student has insurance coverage during Spring quarter, this coverage extends without further required premiums throughout the Summer break.

The UC SHIP is designed to cover approximately 80% of costs associated with catastrophic care. Routine medical care is provided by the Student Health

Service, located on campus. The use of Student Health is essentially free to students (covered through your regular fees), with nominal charges for some lab tests. Please note that the Student Health policies on fees, service provisions, and the insurance plan are subject to change on an annual basis. If you have questions about health insurance and services please contact Student Health Services directly at (805) 893-2592.

Nonresident Tuition

Graduate students who are U.S. citizens or permanent residents, but who are not residents of the State of California at the time of their admission, will have to pay nonresident tuition in addition to the usual fees and charges. It takes one year to establish California residency, and it is your responsibility to do so. You will be provided guidelines and information on steps to take to become a resident of California. You may consult the campus Residence Deputy at (805) 893-3033. As of Fall 2016, the amount of Nonresident Tuition for the academic year is $15,102 (amount subject to change).

Graduate students who are not U.S. citizens or permanent residents will be required to pay nonresident tuition for the duration of their studies. International students' nonresident tuition expenses are waived each quarter after advancement to candidacy for a maximum of nine (9) quarters, provided they have not exceeded the department's normative time for completing the Ph.D. This represents significant savings and strong incentive for international students to complete their coursework and advance as quickly as possible.

UNDERGRADUATE PREPARATION

Students who have taken the following undergraduate courses are usually sufficiently prepared to enter our graduate program:

All students:

Linear Algebra: Math 108 AB

Real Analysis: Math 118 ABC

Complex Analysis: Math 122 AB

And in the case of the M.A. in Mathematics, and Ph.D. students with an interest in pure math:

Modern Algebra: Math 111 ABC

And in the case of the M.A. in Applied Mathematics, and Ph.D. students with an interest in applied math:

Numerical Analysis: Math 104 ABC

Applied Mathematics: Math 124 AB

MASTER OF ARTS PROGRAMS AND REQUIREMENTS

The specific requirements for the Master of Arts in Mathematics and the Master of Arts in Applied Mathematics are listed below. In consultation with a departmental academic advisor, all students are required to create an individual academic plan to satisfy the degree requirements.

An M.A. degree candidate must complete 42 units, 24 of which must be graduate courses in the 200 series offered by the Department of Mathematics. The remaining 18 units may be selected from upper division or graduate courses in mathematics, or in appropriate related fields with the approval of the Mathematics Graduate Committee. A maximum of 8 units of Math 596 and/or Math 598 may be counted toward the degree. All graduate level courses (200 series) must be taken for a letter grade, with a minimum grade point average of B. Other courses may be taken S/U. Approval of the student's advisor and the Graduate Committee is required to utilize Math 260 as part of the 24 graduate level units. Normally a student will complete the M.A. degree in four to six quarters.

At the earliest reasonable time, students enrolled in the Thesis Option should choose a faculty member familiar with their work to serve as their Thesis Advisor. Normally this faculty member would be one of the student's instructors. The thesis may be either an exposition of an area of mathematics needing organization and synthesis or a presentation of original research on a relatively small (but interesting) problem.

Graduate students who expect to complete the requirements for the Master's degree by the end of a given quarter must notify the Staff Graduate Advisor at the beginning of that quarter.

M.A. students interested in continuing on to the Ph.D. may be admitted to the Ph.D. program only by formal petition to the Graduate Committee, with final approval provided by the Graduate Division. Approval is not automatic and will depend upon the student's academic record. These students will normally follow the Examination Option for the Master's degree. To be considered for admission to the Ph.D. program, students are expected to complete their coursework and comprehensive examinations at the academic level that is expected of Ph.D. degree candidates, and should follow the schedule described in the Ph.D. program section of this booklet under the heading, "Normal Progress for Ph.D. Students."

Master of Arts – Mathematics

Degree Requirements

Students who have had some but not all of the requisite material in upper division undergraduate coursework in linear algebra, algebra, real analysis, and complex analysis should complete their work in these areas during their first year by taking the appropriate courses chosen from Math 108 AB, 111 ABC, 118 ABC, and 122 AB.

The department offers two options for completing the degree: thesis or examination. In consultation with a departmental academic advisor, all students are required to create an individual academic plan to satisfy the degree requirements.

Thesis Option: This option requires demonstration of adequate knowledge in linear algebra, modern algebra, real and complex analysis, as well as preparation of an acceptable thesis and oral defense of the thesis before a faculty committee. The 24 graduate units in mathematics must include at least one core full-year graduate course sequence in either Algebra or Analysis.

Before embarking upon a thesis, students should be certain that they have the approval of the faculty member they wish to be their Thesis Advisor. The student and Thesis Advisor must present to the Graduate Committee for its approval a coherent coursework and thesis program no later than the start of their second year. The Thesis Committee will be appointed in accordance with general University policies. All Thesis Committees must have a minimum of three regular (not visiting) faculty. One member, at most, may be from outside the Mathematics department.

The final copies of the thesis are to be prepared in accordance with the requirements given in the document entitled, “Guide to Filing Theses and Dissertations” which can be obtained via the Graduate Division website, http://www.graddiv.ucsb.edu.

Examination Option: Students must satisfy the area requirements for Algebra and Analysis. A student who wishes to substitute a different area requirement for one of the above areas may, with the support of a faculty advisor, petition the Graduate Committee for approval.

Master of Arts – Applied Mathematics

Degree Requirements

Students who have had some but not all of the requisite material in upper division undergraduate coursework in linear algebra, differential equations, real and complex analysis, and numerical analysis should complete their work in these

areas during their first year by taking the appropriate courses chosen from Math 108AB, 124AB, 118ABC, 122AB, and 104ABC.

An M.A. Applied Mathematics student has the option to take up to 9 units of coursework outside the Mathematics Department. These outside units may be applied toward the 42 unit requirement with the approval of the Mathematics Graduate Committee.

The department offers two options for completing the degree: thesis or examination. In consultation with a departmental academic advisor, all students are required to create an individual academic plan to satisfy the degree requirements.

Thesis Option: This option requires demonstration of adequate knowledge in linear algebra, real and complex analysis, numerical analysis, and differential equations, as well as preparation of an acceptable thesis and oral defense of the thesis before a faculty committee. The 24 graduate units in mathematics must include at least one core full-year graduate course sequence in Applied Math.

Before embarking upon a thesis, students should be certain that they have the approval of the faculty member they wish to be their Thesis Advisor. The student and Thesis Advisor must present to the Graduate Committee for its approval a coherent coursework and thesis program no later than the start of their second year. The Thesis Committee will be appointed in accordance with general University policies. All Thesis Committees must have a minimum of three regular (not visiting) faculty. One member, at most, may be from outside the Mathematics department.

The final copies of the thesis are to be prepared in accordance with the requirements given in the document entitled, “Guide to Filing Theses and Dissertations” which can be obtained via the Graduate Division website, http://www.graddiv.ucsb.edu.

Examination Option: Students must satisfy the area requirements in Applied Mathematics and Analysis. A student who wishes to substitute a different area requirement for Analysis areas may, with the support of a faculty advisor, petition the Graduate Committee for approval.

NORMAL PROGRESS FOR M.A. STUDENTS

The ideal course load for the well-prepared first-year M.A. student should consist of three courses per quarter, at least two of which are graduate courses satisfying the course component of two area requirements. It is expected that a student will make sufficient progress to finish the Masters degree in no more than six quarters of full-time study.

PH.D. PROGRAMS AND REQUIREMENTS

The Ph.D. program in Mathematics has the following degree requirements (details follow). In consultation with a departmental academic advisor, all students are required to create an individual academic plan to satisfy these requirements.

1. *Area Requirements:* A core full-year graduate course sequence and comprehensive exam must be passed in each of three areas of mathematics.

2. *Course Requirements:* 72 total units of graduate work offered by the Mathematics department, including a fourth full-year sequence (courses are four units each).

3. *Preliminary Oral Examination and Advancement to Candidacy:* An oral examination administered by the student's Doctoral Committee on topics set by the committee that are close to the area of the student's research. Doctoral students are required to advance to candidacy within four years of beginning their graduate studies at UCSB.

4. *Dissertation:* A dissertation satisfactory to the Doctoral Committee must be written and successfully defended.

Modifications of the requirements for the CSE emphasis and exceptions for students who wish to take exams in the Department of Statistics and Applied Probability appear below.

OPTIONAL GRADUATE DEGREE EMPHASIS IN COMPUTATIONAL SCIENCE AND ENGINEERING

The Departments of Chemical Engineering, Computer Science, Electrical and Computer Engineering, Mathematics, and Mechanical and Environmental Engineering, and Geology offer an interdisciplinary Master's and Ph.D. degree emphasis in Computational Science and Engineering (CSE).

Although CSE includes elements from computer science, applied mathematics, engineering and science, it focuses on the integration of knowledge and methodologies from all of these disciplines and, as such, is a subject distinct from any of them.

All students pursuing an emphasis in CSE must complete the following:

• Numerical Methods: Mathematics 206A-B-C-D (students must take at least three)

• Parallel Computing: Computer Science 240A-B (students must take at least one)

• Applied Mathematics: Students must take a two course sequence from either the Mathematics 243A-B or the Mathematics 246A-B sequence.

The specific requirements for the M.A. in Mathematics (thesis option only) with the CSE emphasis are as follows:

• The completion of the above requirements for an M.A. in mathematics

• A Master's thesis in CSE.

Students pursuing a Ph.D. with an emphasis in CSE must:

• Complete the above requirements for a Ph.D. in mathematics

• Write and defend a dissertation in CSE.

Probability and Statistics

Mathematics Ph.D. students with secondary interests in Probability and/or Statistics may, with the approval of the Graduate Committee, substitute coursework (or area requirements) in these areas. Such students should get course and exam information from the Department of Statistics and Applied Probability and discuss their plans with their advisor.

AREA AND COURSE REQUIREMENTS

The student must satisfy three area requirements. To fulfill an area requirement, typically the student will pass an examination on (primarily) undergraduate material and complete a designated graduate-level sequence with good grades. More detailed descriptions of the area exams appear later. Completion of the three area requirements will ensure the student has passed three full-year graduate course sequences in the Department of Mathematics. *Two pre-approved options are: Algebra, Analysis, and any other, or Analysis, Applied, and Geometry/Topology*. Equivalent sets of requirements may be proposed by a student, in consultation with his/her advisor, for approval by the Graduate Committee.

Students must pass a total of 72 units of 200 level graduate courses offered by the Mathematics department, with a grade of a least B or S in each course; courses fulfilling area requirements normally require an average grade of at least A- for the sequence. The required 72 units of graduate coursework must include a fourth complete full-year graduate sequence, which must also meet the A-average course sequence requirement. S/U grading is not allowed in the four full-year sequences (48 of the 72 total units). Mathematics 596 and/or 599 may not be used to satisfy any portion of the 72 unit requirement. Students must obtain approval of their faculty advisor and the Graduate Committee in order to use any Mathematics 260 courses to satisfy the 72 unit requirement.

DOCTORAL COMMITTEE

Choosing a Dissertation Advisor is not a triviality, but it is usually a natural process. It is the responsibility of the individual student to ask a faculty member of the Mathematics department to serve as his or her Dissertation Advisor. A

faculty member is likely to accept a thesis student who has done well in graduate courses related to that faculty member's field of research. Often the student will take a reading course (Math 596) with the faculty member first in order to ensure mutually acceptable compatibility of interests and expectations. The Dissertation Advisor will help to find other members of the student's Doctoral Committee and will supervise the student's dissertation research.

ORAL EXAMINATION REQUIREMENTS

After fulfilling all of the area requirements, each Ph.D. student shall take and pass the Preliminary Oral Examination given by the student's Doctoral Committee before advancing to candidacy. Students may advance to candidacy before completing the 72 unit requirement, with the condition that they submit to the Graduate Committee a satisfactory proposal of how and when the missing units will be completed.

The purpose of this examination is to ensure that the student has: (A) gained breadth of knowledge on advanced topics in the areas surrounding his/her proposed dissertation topic and (B) engaged in sufficient preliminary reading and research on the proposed dissertation topic to convince his/her Doctoral Committee that he/she has a reasonably good chance to write an acceptable dissertation.

The content of the examination meeting criterion (A) shall be agreed upon in writing by the student and the members of his/her committee approximately three months prior to the proposed date of the examination, and will normally be equivalent to the material in approximately five quarter courses beyond the first year graduate level. Criterion (B) requires a satisfactory presentation by the student of his/her proposed research topic and the results of his/her preliminary reading and research on the topic. The oral examination is scheduled to last two hours – one hour for the presentation, and one hour for questions.

DISSERTATION

After advancing to candidacy, students complete their original research, write their dissertation on a topic agreed upon with their Doctoral Committee, and take any other appropriate courses.

Students complete the requirements for the Ph.D. with the oral dissertation defense to their Doctoral Committee and the formal filing of their dissertation with the Graduate Division. Students must make the arrangements for their dissertation defense. In anything other than exceptional circumstances, these arrangements must be in place at least one month prior to the date of the defense. Such arrangements include, but are not limited to: (i) confirming that all of the committee members are available on the date in question, (ii) providing copies of the dissertation in a reasonably final form to each committee member, and (iii) informing the Staff Graduate Advisor.

NORMAL PROGRESS FOR PH.D. STUDENTS

First Two Years

Most students who have not yet begun a research program should take three courses per quarter, although in some cases the Graduate Committee may deem two or four courses to constitute a reasonable program. The program should be planned in close consultation with a faculty advisor. Ideally, course selections in the first two years should enable the student to complete the area requirements (exams and courses) in three areas, as well as a fourth full-year course sequence. When appropriate, it is acceptable for entering students to take one or more upper division undergraduate courses in their first year to fill gaps in their preparation for graduate level coursework. Students entering with a strong masters degree are expected to move through the program more rapidly and should complete area requirements within one year.

Third Year

By the end of this year the student should know the area in which he or she wants to do research and should have secured a Dissertation Advisor. Ideally, research will begin during this year and the student will advance to candidacy during the year or soon thereafter. To reiterate, doctoral students are required to advance to candidacy within four years of beginning their graduate studies at UCSB.

Fourth Year and Beyond

If all goes well, the student will complete the Ph.D. research, write up the results, successfully defend the dissertation and receive the degree. The official departmental normative time to the Ph.D. degree is 6 years, although many students complete the Ph.D. in 5 years.

AREA REQUIREMENTS FOR M.A. AND PH.D.

Each requirement has two parts:

1. Examination:

The examinations are offered twice a year – during the week before classes begin in the Fall Quarter, and again about two weeks before the end of classes in the Spring Quarter. The examinations are designed to test whether students have adequate knowledge of relevant material, not necessarily whether they have the ability to do research. Normally a grade of A- on the exam will be adequate to pass at the Ph.D. level and a grade of B to pass at the M.A. level. It is advantageous for a Master's student to pass at the Ph.D. level if the student wishes ultimately to apply to the Ph.D. program.

Ph.D. students are required to pass two exams at the Ph.D. level and a third exam at the Master's level or better. Students are allowed unlimited attempts at the exams within certain time constraints (described below), and the department strongly encourages students to attempt at least one exam each time they are offered until they have completed their exam requirements. Any student or admitted applicant may request copies of old exams from the Graduate Program Assistant.

The timeline is as follows: Ph.D. students are *expected* to pass all qualifying exams by the end of their second year and are *required* to pass them by the Spring of their third year. To be eligible for continued financial support the student should obtain at least one Ph.D. pass and a second pass at least the Master's level by the end of the second year.

2. Completion of a designated one-year graduate course with good grades.

Normally, an average grade of A- for Ph.D. students and of B for Master's students will be adequate to satisfy part two of an area requirement. Courses taken S/U will not satisfy the requirement. Whether or not a student has fulfilled an area requirement will be determined by the Graduate Committee in consultation with the student's graduate course instructors and the faculty members who prepared and graded the examination in that area. The total performance in the examination and coursework will determine whether or not the area requirement is satisfied.

The Graduate Committee will determine how any deficiencies, if present, may be corrected. Complete descriptions of the various area requirements follow. Students who have successfully completed graduate level coursework at another institution can petition the Graduate Committee to allow that coursework to satisfy the course component of an area requirement.

AREA REQUIREMENT IN ALGEBRA

This requirement consists of 1) an examination based on undergraduate material such as that found in UCSB courses 108 AB and 111 ABC, and 2) a one year graduate course in Modern Algebra: Math 220 ABC.

Algebra Examination

The algebra exam consists of three parts: a section on groups, a section on rings and fields, and a section on linear algebra. A handout describing the algebra exam in more detail is under development and will be available from Graduate Program Assistant once it has been completed. The examination will permit a limited selection in the choice of questions to answer. Topics include:

Groups: Lagrange's theorem, permutation groups, Cayley's theorem, cyclic groups, morphisms, quotient groups, automorphism groups, direct product representation for finitely generated abelian groups, Sylow theorems, groups of small order.

Rings and Fields: Integer and polynomial rings, factorization theory, subrings and ideals, morphisms, quotient rings, fields of quotients, Euclidean rings, prime and maximal ideals, algebraic and transcendental field extensions, prime fields, finite fields, Galois theory over the rational numbers.

Linear Algebra: Vector spaces, linear transformations, matrices, system of linear equations, eigenvalues and eigenvectors, inner product spaces, normal linear transformations, similarity, elementary divisors and invariant factors, canonical forms.

References:

1. Herstein, I., *Topics in Algebra*, third edition
2. Jones, Burton W., *An Introduction to Modern Algebra*
3. Stewart, I., *Galois Theory*
4. Birkhoff, G., Maclane, D., *A Survey of Modern Algebra*
5. Strang, G., *Linear Algebra and its Applications*
6. Halmos, P., *Finite Dimensional Vector Spaces*
7. McCoy, N., *Introduction to Modern Algebra*
8. Hoffman, K., Kunze, R., *Linear Algebra*
9. Gantmacher, F.R., *The Theory of Matrices, Vol. I*

AREA REQUIREMENT IN ANALYSIS

This requirement consists of 1) an examination based on undergraduate material such as that found in UCSB courses 118 ABC and 122AB, and 2) one of the following graduate course sequences:

Real Analysis: Math 201 ABC

Complex Analysis: Math 202 ABC

Real and Complex: Math 201 AB and Math 202 AB

Students should consult with their advisor to decide which of these three options would be best.

Analysis Examination

The analysis exam consists of two parts, a section on real analysis and a section on complex analysis. A handout describing the analysis exam in more detail is posted on the Mathematics Department web site.

Topics in Real Analysis: The real number system, topology of R^n, continuity, differentiability, Riemann integration, sequences and series, convergence processes including uniform convergence, functions of several variables, and introductions to metric spaces and to measure and integration.

Topics in Complex Analysis: Cauchy-Riemann Equations,complex integration, Cauchy's Theorem, Cauchy's Integral formulas, infinite series (Taylor and Laurent), residue theorem and evaluations of integrals, conformal mapping.

References:

All of the topics on the exam are covered in Walter Rudin's *Principles of Mathematical Analysis*. The book contains more than is required. The exam excludes 7.19-7.33, 8.15-8.22, and all of Chapters 10 and 11. From Chapter 6 the exam will cover only ordinary Riemann integration, not the more general Riemann-Stieltjes integral. Many other analysis books contain most or all of the material.

References:

All of the material covered in the exam is contained in Brown and Churchill's *Complex Variables and Applications*, 8th edition, sections 1-80. (A cheaper book, Murray Spiegel's Schaum's Outline book, *Complex Variables*, contains these topics as well, and has many solved problems.)

AREA REQUIREMENT IN APPLIED MATHEMATICS

This requirement consists of 1) an examination in either Numerical Analysis or Differential Equations, based on undergraduate material such as that found in UCSB courses 104 ABC and 124 AB, and 2) a one-year graduate course taken from the following list:

Ordinary Differential Equations: Math 243 ABC

Partial Differential Equations: Math 246 ABC

Numerical Analysis: Math 206 ABCD (only 3 quarters are required)

Numerical Analysis Examination

1. Root Finding, Interpolation and Numerical Integration

• Solution of Nonlinear Equations: Fixed-point iteration, Newton's Method and steepest descent.

• Interpolation: Lagrange polynomials, Newton's divided differences and Neville's algorithm.

• Numerical Integration and Differentiation: Newton-Cotes quadratures and composite rules. Richardson extrapolation. Finite differences.

2. Numerical Linear Algebra

• Direct methods: Gaussian elimination, LU, LDL, and Cholesky factorizations.

• Iterative methods: Jacobi, Gauss-Seidel and SOR.

• The conjugate gradient method.

• Convergence of iterative methods. Condition number of a matrix.

• Approximation of eigenvalues: Gerschgorin circle theorem, the power method, the QR algorithm.

3. Numerical Methods for Ordinary Differential Equations

• Basic existence theory for ODEs: existence, uniqueness, and well-posedness.

• One-step difference methods: Euler's method and Runge-Kutta methods. Consistency and order of one-step methods.

• Linear multi-step methods: Consistency, order, zero-stability (root condition), and convergence. Adams-Bashforth, Adams-Moulton, and predictor-corrector methods.

• Absolute stability.

• Finite difference methods for linear and nonlinear boundary-value problems for ODEs.

4. Finite Difference Methods for PDE's

• Order of accuracy and consistency. The CFL condition.

• Von Neumann analysis and stability.

• Finite difference methods for hyperbolic, elliptic, and parabolic PDEs. Implicit and explicit methods.

5. Approximation Theory

• Least squares approximation.

• Orthogonal polynomials. Chebyshev polynomials and trigonometric approximation.

• The fast Fourier transform.

References:

Numerical Analysis, by David Kincaid and Ward Cheney.

Introduction to Numerical Analysis, by J. Stoer and R. Bulirsch.

Finite Difference Methods for Ordinary and Partial Differential Equations, by Randall J. LeVeque.

Computational Methods in Ordinary Differential Equations, by J.D. Lambert.

Finite Difference Schemes and Partial Differential Equations, by J. C.Strikwerda.

Differential Equations Examination

1. Ordinary Differential Equations

• Existence and uniqueness for the initial value problem

• Continuous dependence on initial conditions

• Linear systems: constant coefficients, periodic systems, fundamental matrices

• Variation of parameters formula

• Phase plane analysis

• Stability of critical points

• Planar autonomous systems

2. Partial Differential Equations

• Well-posedness

• The wave equation: D'Alembert's formula, energy, method of characteristics, finite propagation speed, weak solutions

• Laplace equation: Green's identities, Poisson kernel, maximum principle, harmonic functions

• Heat equation: Fourier transform, maximum principle, dissipation

• Boundary value problems via Fourier series

• Eigenvalue problems via minimization

•The method of characteristics for first order equations

References:

Differential Equations, Dynamical Systems, and Linear Algebra, by Hirsch and Smale.

Partial Differential Equations, an Introduction, by Strauss.

AREA REQUIREMENT IN GEOMETRY/TOPOLOGY

This requirement consists of 1) an examination in either Geometry or Topology, based on undergraduate and graduate material such as that found in UCSB courses 147 AB and 240 AB (Geometry), or 145 and 221 A (Topology), and 2) a one-year graduate course taken from the following list:

Differential Geometry and Riemannian Geometry: Math 240 ABC

Foundations of Topology, Manifolds, Homotopy Theory: Math 221 ABC

Geometry Examination

Arc length of curves, curvature and torsion, surfaces, first and second fundamental forms, curvature of surfaces, geodesics, Gauss-Bonnet formula.

References:

1. do Carmo, *Differential Geometry of Curves and Surfaces*, Chapters 1–4

2. McCleary, *Geometry from a Differential Viewpoint*

Topology Examination

Metric spaces, topological spaces, continuous functions, product spaces, compactness, connectedness, path-connectedness, completeness, quotient spaces, fundamental group, covering spaces.

References*:*

1. Croom, *Principles of Topology*

2. Sutherland, *Introduction to Metric and Topological Spaces*

3. Kahn, *Topology, Introduction to the Point-Set and Algebraic Areas*

4. Munkres, *Topology: A First Course*

5. Hatcher, *Algebraic Topology*

The University of California, in accordance with applicable Federal and State law and University policy, does not discriminate on the basis of race, color, national origin, religion, sex, disability, age, medical condition (cancer-related), ancestry, marital status, citizenship, sexual orientation, or status as a Vietnam-era veteran or special disabled veteran. The University also prohibits sexual harassment. This nondiscrimination policy covers admission, access, and treatment in University programs and activities. Inquiries regarding the University's student-related nondiscrimination policies may be directed to: The Office of Equal Opportunity and Sexual Harassment/Title IX Compliance, (805) 893-2701.

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