2018-19 GRADUATE COURSE DESCRIPTIONS

MATH 201 A-B-C (FWS), Akemann/Birnir, *Real Analysis*
Measure theory and integration. Point set topology. Principles of functional analysis. L^p spaces. The Riesz representation theorem. Topics in real and functional analysis.

MATH 202 A-B-C (FWS), Labutin/Putinar, *Complex Analysis*
Analytic functions. Complex integration. Cauchy's theorem. Series and product developments. Entire functions. Conformal mappings. Topics in complex analysis.

MATH 206 A (F), Chandrasekaran, *Matrix Analysis & Computation*
Graduate level-matrix theory with introduction to matrix computations. SVDs, pseudoinverses, variational characterization of eigenvalues, perturbation theory, direct and interative methods for matrix computations.

MATH 206 B (W), Petzold, *Numerical Simulation*
Linear multistep methods and Runge-Kutta methods for ordinary differential equations: stability, order and convergence. Stiffness. Differential algebraic equations. Numerical solution of boundary value problems.

MATH 206 C (S), Ceniceros, *Numerical Solution of Partial Differential Equations - Finite Difference Methods*
Finite difference methods for hyperbolic, parabolic and elliptic PDEs, with application to problems in science and engineering. Convergence, consistency, order and stability of finite difference methods. Dissipation and dispersion. Finite volume methods. Software design and adaptivity.

MATH 206 D (F), H. Zhou, *Numerical Solution of Partial Differential Equations - Finite Element Methods*
Weighted residual and finite element methods for the solution of hyperbolic, parabolic and elliptical partial differential equations, with application to problems in science and engineering. Error estimates. Standard and discontinuous Galerkin methods.

MATH 220 A-B-C (FWS), Goodearl, *Modern Algebra*
Group theory, ring and module theory, field theory, Galois theory, other topics.

MATH 221 A (F), Cooper, *Foundations of Topology*
Metric spaces, topological spaces, continuity, Hausdorff condition, compactness, connectedness, product spaces, quotient spaces. Other topics as time allows.

MATH 221 B (W), Bigelow, *Homotopy Theory*
Homotopy groups, exact sequences, fiber spaces, covering spaces, van Kampen Theorem.

MATH 221 C (S), Bigelow, *Differential Topology*
Topological manifolds, differentiable manifolds, transversality, tangent bundles, Borsuk-Ulam theorem, orientation and intersection number, Lefschetz fixed point theorem, vector fields.

MATH 225 A-B-C (FWS), Agboola, *Topics in Number Theory*
The prerequisites for this course are a solid knowledge of the basic first-year graduate courses in algebra and analysis, and a level of mathematical maturity appropriate for an advanced graduate course.

This course is intended to be a year-long introduction to number theory and certain topics in arithmetic geometry.

The first quarter of the sequence will be devoted to an introduction to algebraic number theory. A list of topics that will be covered includes: Basic commutative algebra: Noetherian properties, integrality, rings of integers. More commutative algebra: Dedekind domains, unique factorisation of ideals, localisation. Norms, traces and discriminants. Decomposition of prime ideals in an extension field. Class numbers and units. Finiteness of the class number: Minkowski bounds. Dirichlet's unit theorem. Explicit calculation of units. Decomposition of prime ideals revisited: the decomposition group and the inertia group associated to a prime ideal. A nice proof of quadratic reciprocity.

The second quarter of the sequence will consist of an introduction to the theory of elliptic curves. Topics that will be covered include: Algebraic varieties and algebraic curves. The geometry of elliptic curves. Elliptic curves over finite and local fields. The Mordell-Weil group and the conjecture of Birch and Swinnerton-Dyer.

In the third and final quarter, we shall discuss further topics concerning elliptic curves, modular forms, and class field theory.

Some references:

For the first quarter:

"Algebraic Theory of Numbers", by P. Samuel (recently reprinted as a Dover paperback).

"Algebraic Number Theory" by A. Frohlich and M. J. Taylor (CUP).

"Algebraic Number Theory", by S. Lang (Springer).

For the second quarter:

"The arithmetic of elliptic curves", by J. Silverman (Springer).

"Advanced topics in the arithmetic of elliptic curves", by J. Silverman (Springer)

"The arithmetic of elliptic curves", by J. Tate, Invent. Math. 23 (1974), 179--206.

For the third quarter:

"Rational points on modular elliptic curves", by H. Darmon (AMS) (also available for free from Darmon's webpage at McGill).

"The p-adic upper half-plane", by S. Dasgupta and J. Teitelbaum (available for free from Samit Dasgupta's webpage at UC Santa Cruz).

"Local fields", by J.-P. Serre (Springer)

"Automorphic forms and representations", by D. Bump (CUP).

MATH 227 A (F), Long, *Advanced Topics in Geometric and Algebraic Topology*
The course will start with an introduction to hyperbolic geometry, leading to some other topic towards the end of the quarter. This latter topic may be guided somewhat by the interests of the audience.

MATH 227 B (W), Cooper, *Advanced Topics in Geometric and Algebraic Topology*
This course will cover various topics in low dimensional topology and geometric structures on manifolds including some of the following: Hyperbolic geometry, Teichmuller Space, measured foliations, Thurston compactification. 3-Manifolds, geometrization theorem, knot theory. Projective geometry, convex projective structures, geometric transitions. Higher Teichmuller theory, Fock-Goncharov coordinates, mixed structures, a Thurstontype compactification. I will point out some of the many intriguing open questions in these areas.

MATH 227C (S), Wang, *Advanced Topics in Geometric and Algebraic Topology*
From (2+1)-TQFTs to modular forms Description: A (2+1)-TQFT, essentially the same as a modular tensor category algebraically, provides representations of all mapping class groups (MCGs), in particular the MCG of the torus SL(2,Z). A deep theorem says that the kernel of such a TQFT representation of SL(2,Z) is always a congruence subgroup. Congruence subgroups are closely related to modular forms, which are generalizations of the modular function j=q^-1+744+196884q+...appeared in the monster moonshine. We will start with an introduction to such TQFT representations of SL(2,Z), and then discuss the preliminary program of attaching vector-valued modular forms to modular tensor categories with some extra data.

MATH 232 A (W), McCammond, *Algebraic Topology*
Singular homology and cohomology, exact sequences, Hurewicz theorem, Poincare duality.

MATH 236 A-B (FW), Huisgen-Zimmermann, *Homological Algebra*
Algebraic construction of homology and cohomology theories, aimed at applications to topology, geometry, groups and rings. Special emphasis on hom and tensor functors; projective, injective and flat modules; exact sequences; chain complexes; derived functors, in particular, ext and tor.

MATH 240 A-B-C (FWS), Dai/Wei/Ye, *Introduction to Differential Geometry and Riemannian Geometry*
Topics include geometry of surfaces, manifolds, differential forms, Lie groups, Riemannian manifolds, Levi-Civita connection and curvature, curvature and topology, Hodge theory. Additional topics such as bundles and characteristic classes, spin structures Dirac operator, comparison theorems in Riemannian geometry.

MATH 241 A (F), Ye, *Topics in Differential Geometry*

MATH 241 B (W), Dai, *Topics in Differential Geometry*
Index Theorems, Positive Scalar Curvature, and Positive Mass Theorems

Scalar curvature is the weakest of the three classical curvature concept. Thus it is fascinating that scalar curvature connects with a wide range of remarkable developments in mathematics and physics.
In this course we will tell part of the story, focusing on its connection with the Atiyah-Singer index theorems and the general relativity.

MATH 241 C (S), Wei, *Topics in Differential Geometry*
Riemmannian Metric Measure Spaces prerequisite: 240AB or consent of instructor

Just as limits of differentiable functions may not be differentiable, limits of smooth manifolds may not be smooth or even (topological) manifold. Can we talk about curvature bounds for non-smooth spaces? The notion of sectional curvature lower (or upper) bound can be defined on very general metric spaces using triangle comparison. The corresponding question for Ricci curvature is much harder. After many workds recently people find that Riemannian metric measure space, referred as RCD(K,N), is the right object. The class includes manifolds with lower Ricci curavture and their Gromov-Hausdorff limits. Many of the results for smooth manifolds with lower Ricci curvature bounds can be extended to RCD spaces. This is a very active area of research. It relates to optimal transport, convex geometry, metric geometry. The goal of the course is to introduce and study RCD spaces, in particular some topological and rigidity results.

MATH 246 A-B-C (FWS) Harutyunyan/H. Zhou, *Partial Differential Equations*
First-order nonlinear equations; the Cauchy problem, elements of distribution theory an Sobolev spaces; the heat, wave, and Laplace equations; additional topics such as quasilinear symmetric hyperbolic systems, elliptic regularity theory.

MATH 260AA (W), Harutyunyan, *Calculus of Variations*
The Calculus of Variations deals with minimization (or maximization) of an integral functional. A model problem reads as follows: Let n;N 2 $\mathbf{R}_n$ and let _ $\mathbf{R}_n$ be a bounded open set. Assume the function L: _ $\mathbf{R}_N$ _ $\mathbf{R}_{N\_n}$! **R** is smooth. We want to minimize the value of the integral functional Z L(x;u(x);ru(x))dx over the set of all functions u 2 C_1() such that u(x) = u_0(x) for x 2 @; where u_0 is a given continuous function.
A classical example is the so called Fermat principle for light, which is the following: We want to find the trajectory that should follow a light ray in a medium with non-constant refraction index. The problem can be formulated within the above framework, namely one has n = N = 1 and the function L (that is called the Lagrangian) will be given by the formula
L(x; u; _) = g(x; u)
p
1 + $\_2$;
where the variable _ plays the role of the gradient. The minimization problem is then to minimize the integral functional
Z b
a
g(x; u(x))
p
1 + g_{02}(x)dx
under the initial and end conditions u(a) = _ and u(b) = _; where a; b; _ and _ are given constants.
Modern Calculus of Variations has tight connections with Partial Differential Equations and existence and uniqueness of their solutions, Continuum Mechanics (Elasticity and Plasticity), Fracture Mechanics, Composite Materials, Material Science, Differential Geometry, etc.
The tentative contents are below: (note that each title contains many subtitles that are not listed below)
1. One dimensional calculus of variations, existence and uniqueness of minimizers, nonexistence.
2. Multi-dimensional calculus of variations, the Euler-Lagrange equation and Null-Lagrangians.
3. Existence and uniqueness for convex and strictly convex problems, nonexistence.
4. The notions of polyconvexity, quasiconvexity and rank-one convexity.
5. The direct method in the calculus of variations, lower-semicontinuity of integrals.

6. The relation between existence and quasiconvexity.
7. Regularity of minimizers.
8. Relaxation and envelopes.
9. -convergence and basic properties

The lecture material will be covered by the below literature.

1. Introduction to the Calculus of Variations, by B. Dacorogna (book).
2. The Direct Method in the Calculus of Variations, by B. Dacorogna (book).
3. In introduction to Gamma convergence, G. Dal-Maso (book).
4. Partial Differential Equation, by L. C. Evans (lecture notes).
5. Weak Convergence Methods for Nonlinear Partial Differential Equations, L. C. Evans (book).
6. An Introduction to Variational Inequalities and Their Applications, by D. Kinderlehrer and G. Stampaccia (book).

MATH 260EE (FWS), Cooper, *Graduate Student Colloquium*

Topics in algebra, analysis, applied mathematics, combinatorial mathematics, functional analysis, geometry, statistics, topology, by means of lectures and informal conferences with members of faculty.

MATH 260H (F) H. Zhou, *Introduction to Inverse Problems*

In inverse problems one attempts to determine the interior properties of a medium by applying various non-intrusive methods. The mathematical problems under study are often motivated by real world application purposes, including questions arising in medical imaging (e.g. CT, EIT, MRI), geophysics (seismology), mathematical physics, etc. In this topics course, we will introduce the mathematical analysis of some basic types of inverse problems, including X-ray and Radon transforms, inverse boundary value problems (e.g. the inverse conductivity problem), and their generalizations in non-trivial geometry (e.g. anisotropic media). If time permits, we will also give brief introduction to related inverse problems, such as travel time tomography, coupled-physics inverse problems, invisibility and cloaking.

References:

[1] Lecture notes on Calderon problem, Mikko Salo, Spring 2008.
[2] Inverse problems course notes, Gunther Uhlmann (notes taken by Rolfe Schmidt), Fall 2009.
[3] Inverse problems: visibility and invisibility, Gunther Uhlmann, JEDP 2012.

MATH 260J (F) Atzberger, *Machine Learning: Foundations and Applications*

This special topics course will survey current approaches in machine learning, their mathematical foundations, and practical computational methods. Recent advances in measurement and scientific simulation are resulting in a plethora of data. This presents a number of challenges both for the design of high-throughput experiments/simulations and for the interpretation of the resulting large sets of data. Advances in combining approaches, such as Bayesian statistics, with large-scale scientific computation are resulted in new methodologies and algorithms for inferring information from such data. In this special topics class, we will give a survey of current Machine Learning techniques and practical methods that draw on results from stochastic analysis. The beginning introductory materials of the course will use the books *"The Elements of Statistical Learning: Data Mining, Inference, and Prediction"* by Hastie, Tibshirani, Friedman and *"Stochastic Differential Equations: An Introduction with Applications,"* by Bernt Øksendal. The remaining part of the course will be based on special lecture materials and recent papers in the literature.

More details concerning the specific topics can be found below.

Sample of Topics:

Introduction

o Historic Motivations.

o Bayesian vs classical statistics.
o Maximum likelihood methods.
o Uncertainty quantification.
o Markov-chain Monte-Carlo sampling.
o Motivating applications from the sciences, engineering, and finance.

🎬*Introduction to Stochastic Processes*
o Random Walks: Lattice Process, Poisson Process, Markov-Chain.
o Brownian Motion, Martingales, Ito Process.
o Sigma-Algebras, Filtration of Processes, Conditional Expectation.
o Stochastic Differential Equations (SDEs), Ito Integral, Ito's Formula.
o Stochastic Finite Difference Methods and Finite Element Methods for SPDEs.
o Karhunen–Loève Theorem and Expansions.
o Wiener Chaos Expansion.
o SDEs connection to PDEs, Infinitesimal Generator, Forward and Backward Kolomogorov PDEs.
o Stochastic Methods for Analysis of Elliptic and Parabolic PDEs.

🎬*Statistical Inference and Machine Learning*
o Supervised Learning Methods
o Linear Methods for Regression and Classification
o Kernel smoothing methods
o Parametric vs non-parametric regression
o Model selection and bias-variance trade-offs
o Neural Networks, Support Vector Machines, Graphical Models.

🎬*Numerical Methods for Machine Learning*
o Sparse Matrix Methods
o Preconditioners and Iterative Methods
o Conjugate Gradient Methods
o Non-linear Optimization Methods
o Markov-Chain Monte-Carlo Sampling
o Sampling with stochastic processes
o Dimension reduction (graph Laplacian, eigen-analysis)
o Stochastic averaging and multiscale analysis.

References:
1. *The Elements of Statistical Learning Data Mining, Inference, and Prediction,* T. Hastie, R. Tibshirani, J. Friedman, (2013).
2. *Machine Learning: A Probabilistic Perspective*, K. P. Murphy, (2012).
3. *Stochastic Differential Equations: An Introduction with Applications,* Oksendal, B. K., (2003).
4. *Handbook of Stochastic Methods: for Physics, Chemistry and the Natural Sciences,* Gardiner, C., (2009).

MATH 260Q (F), Jacob, *The Algebraic Theory of Quadratic Forms*
Prerequisite: Math 220C
This will be an introduction to the algebraic theory of quadratic forms. The tools developed in the course are of interest in algebra and number theory.

Let F be a field. We consider the isometry classes of anisotropic, nonsingular, finite-dimensional quadratic forms over F. Here, quadratic forms are given by homogeneous quadratic polynomials, nonsingular means the symmetric matrix associated to the polynomial is nonsingular, and anisotropic means the form does not represent 0 nontrivially. This set of isometry classes can be made into a ring, called the Witt ring of F, where addition is given by the direct sum, multiplication is given by the tensor product, and one extracts the unique anisotropic part of a sum or product in case they are isotropic.

A main goal of the algebraic theory of quadratic forms is to compute the Witt ring of fields. One quickly finds that the Witt ring of the real number is Z and the Witt ring of the complex numbers is Z/2Z. In the course we will develop the tools to compute the Witt rings of the local and global fields of algebraic number theory (including background on these fields). We will also look at connections between the Witt ring of a field and the orderings of the field, including results of interest in real algebraic geometry. Other applications will be to problems involving the theory of finite-dimensional division algebras.

MATH 501 (F), Garfield, *Teaching Assistant Training*
Consideration of ideas about the process of learning mathematics and discussion of approaches to teaching.